Nonlinear Motion of an Air-Cushion Vehicle over Waves

L.J.Doctors*
University of New South Wales, Kensington, Australia

The nonlinear behavior of an air-cushion vehicle (ACV) during coupled pitch and heave motion over a sinusoidal wave is studied. The vehicle is of the plenum-chamber type with a transverse stability skirt, which is a contributor to the nonlinearity. Further nonlinearity results from the contact of the flexible skirt with the surface. The airflow is considered incompressible, and water compliance is neglected. Numerical computations have been made for the craft response in head seas as a function of encounter frequency for a set of Froude numbers and skirt configurations. The motion is found to be highly damped, and strongly dependent on the depth of the stability skirt, though not on its longitudinal position. Weak peaks in the response curves are displayed, their strength and location being a function of the wave height. The effect of skirt contact forces is found to be negligible on the craft motion. The predicted pressure fluctuations can be large and, at sufficiently high frequencies, result in subatmospheric pressures in the cushion.

Nomenclature

A	= discharge area
B	= beam of craft measured at skirt
c	= forward velocity of craft
C	= coefficient
e	= skirt hem clearance
F	= Froude number $= c/(gL)^{1/2}$
g	= acceleration of gravity
h	= height
I	= second moment of mass of craft about transverse axis through the c.g.
k	= wave number
$\mathcal{L}$	= distance measured around skirt hem
L	= length of craft measured at skirt
m	= mass of craft
M	= moment
p	= pressure
Q	= volume flow
R	= reaction
S	= cushion planform area
t	= time
x	= instantaneous longitudinal position of craft
z	= instantaneous height of the craft center of gravity above datum
α	= local wave slope, or amplitude of wave slope
β	= angle between skirt hem and x axis, as seen in plan view, i.e., $\cos\beta = dx/d\mathcal{L}$.
ϵ	= longitudinal position of transverse skirt, ahead of midpoint, as a fraction of L
η	= skirt force deflection constant
θ	= craft pitch angle
μ	= coefficient of friction between skirt and the operating surface
ρ	= density
ω	= wave encounter radian frequency

Subscripts

a	= air
d	= discharge
e	= escape
f	= fan
g	= center of gravity
h	= horizontal
i	= inlet
k	= skirt
max	= maximum
min	= minimum
rms	= root mean square [and multiplied by $(2)^{1/2}$]
r	= craft
v	= vertical
w	= wave
1,2,3	= forward, aft, and stability regions

Superscripts

-	= average
·	$= \dfrac{\partial}{\partial t}$
*	= indicates a variable related to its mean value during the craft motion, divided by h_w, and nondimensionalized with L, g and ρ. However, the craft pitch angle is divided by α_w instead.

Received April 22, 1974; revision received November 7, 1974.
Index categories: Ground (or Water-Surface) Effect Machines; Marine Vessel Trajectories, Stability, and Control.

*Lecturer, School of Mechanical & Industrial Engineering.

Introduction

Previous Work

THE motion of an air-cushion vehicle (ACV) was studied by Swaan and Wahab.[1] They conducted towing-tank tests on a peripheral-jet model and tested it both with, and without, flexible skirts. The model was run in regular waves of various direction. The curves which displayed craft heave and pitch response as a function of forward speed (for a particular wavelength) were fairly flat indicating a high level of damping. Results were also presented for wave heights varying up to about 0.10 of the craft length, and these indicated some nonlinearity in that the response was not proportional to the height of the waves.

Dyne[2] restricted his model tests (also on a peripheral-jet craft) to regular head seas. The air cushion was compartmented into six sections. The model was excited on a flat surface and the stiffness and damping factors were measured and found to be in agreement with his theory. From these results, a prediction for the motion over a regular wavy surface was made using a linear theory.

Van den Brug[3] and Van den Brug and Van Staveren[4] studies were also model tests on an essentially plenum chamber design. Forced oscillation tests in heave, pitch, and roll above ground without forward velocity, and above water with forward velocity were carried out with a view to determine the coefficients in the linearized equations of motion. The manner in which these coefficients vary with forward speed and encounter frequency was displayed graphically. In particular, the added mass of the water was found to be negative—in sharp contradistinction with a ship, where it is positive and of the order of the ship mass.

A method of "equivalent linearization" was suggested by Murthy[5] for the determination of the response spectrum of an ACV traveling in an irregular wind-generated seaway. He gave an example of the technique which requires a knowledge of the craft response to a regular wave.

The practical operation of ACVs in realistic waves has been entirely due to the development of the flexible skirt and a review of the history of this work was given by Jones.[6] Jones quoted model tests on a skirted annular-jet craft which indicated that the damping ratio in heave was approximately 0.5, and for the pitch motion was somewhat lower, being about 0.25. Some results were also given for a craft fitted with finger skirts. These indicated a higher heave damping ratio (in the region of 0.75) compared to the annular-jet skirt case. Thus the craft motion was measurably improved by the fitting of finger skirts. Jones also noted that the unit response was less in higher waves, and that this would be a measure of the nonlinearity in the system.

In the theoretical field, Lin[7] computed the dynamic pitch and heave behavior of an unskirted annular-jet craft fitted with a transverse stability jet. The equations of motion were linearized. Comparisons were made with models and good agreement was found, particularly as the natural frequencies and general magnitude of the pitch and heave response were well predicted. The spring constants for both the static and dynamic cases for a peripheral-jet ACV executing a simple heaving motion were calculated by Ozawa[8] and then used to predict the linear heave motion due to a sinusoidal input. Hogben[9] in his review of the behavior of ACVs over water, also gave the basic theories for simple heave motion, and linearized these for both annular-jet and plenum-chamber craft. In addition, he covered various empirical approaches in his paper.

The heaving response of an ACV moving over regular waves was studied by Reynolds[10] and was later extended to include pitching by Reynolds, West, and Brooks.[11] In both cases the equations of motion were linearized. In the latter paper, the plenum-chamber vehicle considered (as in Fig. 1) had a single lift fan feeding an intermediate chamber (corresponding to the ducting and flexible supply bag utilized by many ACVs). The air was then fed to the fore and aft compartments of the cushion, which had a transverse stability skirt. The incompressible Bernoulli equation and the usual equations of continuity were used in this analysis, and the

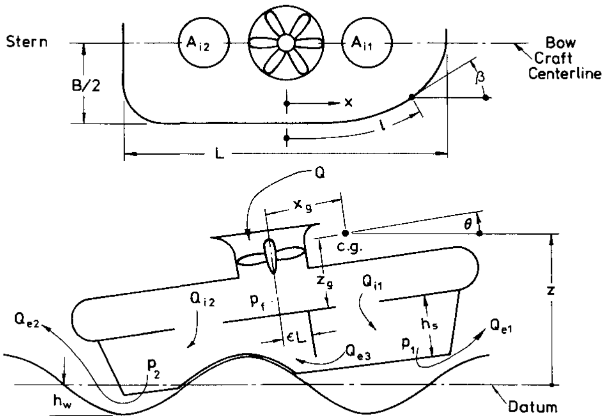

Fig. 1 Layout of craft and notation.

coupled pitch and heave motion for a particular vehicle was computed as a function of the ratio of craft length to wave length.

One feature of the problem which renders this linearization questionable is that the daylight clearance under the skirt is usually quite small, so that skirt contact and deflection occurs in most situations, and this is ignored—as pointed out by Reynolds et al. This would be a very strong nonlinear effect. A second point of concern is related to the cross flow under the transvere stability skirt. The pressure drop across the skirt is proportional to the square of the flow and has the same sign. Consequently, the linearization process is only valid if there is a cross flow in the equilibrium condition, and if the changes during motion are small compared to it. In fact, the equilibrium cross flow itself is usually very small in practice, (the equilibrium pressures in the fore and aft compartment are virtually identical) and the changes in cross flow during the heave and pitch cycle swamp it entirely, so that this physical mechanism is essentially nonlinear to the first order.

Nonlinear heaving motion of a plenum chamber ACV was studied by Yamamoto.[12] He included the effect of the compressibility of air. Other workers have also considered the compressibility factor during simple heaving. These include Leatherwood, Dixon and Stephens[13], Guienne,[14] Leatherwood,[15] and Genin, Ginsberg, and Ting.[16] The basic interest of the last four references lies in their application to high-speed tracked ACVs. Under the conditions of operation projected for these vehicles, the technique of linearization should be valid, since no vehicle contact with the track will be permitted. Incidentally, all these reports which considered the compressibility effect (except the last one), assumed the air within the cushion obeys the polytropic process. However, the *incompressible* Bernoulli equation for discharging to the atmosphere was used. Possibly, this assumption is satisfactory in practice.

Coupled pitch and heave motion of a multiple skirt ACV was analysed by Bickford and Olson,[17] utilizing the assumptions discussed in the preceding paragraph, and analyzing the resulting equations on an analog computer. Other aspects of the mechanics of ACV motion over waves were considered by Breslin,[18] Richardson,[19] and Trillo.[20] It was noted that if compressibility is to be represented correctly on a model ACV, then the atmospheric pressure must be scaled accordingly.

Present Aims

The basic intention in the present work is to study the effect of nonlinearity on the coupled pitch and heave motion of an ACV over waves. As indicated in the previous section, it was felt that the nonlinear behavior of the cross flow under the stability skirt, and the behavior of general skirt contact, should play a major role in determining the motion of the vehicle.

For simplicity, the air will be assumed to be incompressible. Furthermore, the compliance of the water surface, which would affect the stiffness, damping, and mass of the system, will be neglected. One would expect the hydrodynamic effect to be less important at high wave-encounter frequencies. On the other hand, compressibility is likely to be less important at lower frequencies. It is by no means clear under what conditions these restrictions would be unimportant. This question would be answered by a more complete theory.

Theoretical Model

Definition of Vehicle

The craft to be considered is shown in Fig. 1. The center of gravity is located at x_g, z_g relative to the longitudinal midpoint, and above the base from which the skirt is suspended. The ACV executes planar motion, so that its position is defined by the two variables z and θ, the height of the center of gravity above the ground datum and the pitch angle, respectively. The craft has a mass m and the second moment of mass about the transverse axis through the center of gravity is I. The air for the lift system is supplied by one or more fans which discharge into an intermediate settling chamber. In practice this chamber includes the ducting and peripheral bag. The air then flows into the forward and aft cushions through the equivalent areas A_{i1} and A_{i2}, respectively.

Various types of lift fan system can be modeled. These are discussed, for example, in the ASHRAE *Guide and Data Book.*[21] For generality, we will assume the characteristics of the inlet, fan, and settling chamber are defined by

$$p_f = C_{f1} + C_{f2}Q + C_{f3}Q^2 \tag{1}$$

where p_f is the total pressure in the settling chamber, and Q is the total airflow. The discharges into the two cushions and under the skirts are assumed to be given by an application of Bernoulli's equation with an appropriate discharge coefficient, C_d. Hence

$$Q_{i_1} = \mathrm{sgn}(p_f - p_1) A_{i_1} C_{di_1} (2|p_f - p_1|/\rho_a)^{1/2} \tag{2}$$

$$Q_{i_2} = \mathrm{sgn}(p_f - p_2) A_{i_2} C_{di_2} (2|p_f - p_2|/\rho_a)^{1/2} \tag{3}$$

$$Q_{e_1} = \mathrm{sgn}(p_1) A_{e_1} C_{de_1} (2|p_1|/\rho_a)^{1/2} \tag{4}$$

$$Q_{e_2} = \mathrm{sgn}(p_2) A_{e_2} C_{de_2} (2|p_2|/\rho_a)^{1/2} \tag{5}$$

and

$$Q_{e_3} = sgn(p_1 - p_2) A_{e_3} C_{de_3} (2|p_1 - p_2|/\rho_a)^{1/2} \tag{6}$$

Here, the Q_i are the flow rates entering the cushions, and the Q_e are those discharging from the cushions to atmosphere, except that Q_{e3} is the cross flow from the forward to the aft cushion. The air density is ρ_a. While it is clear that Q_{e3} can have either sign, the other four flow rates would normally be only positive. However, it was found that even they can be negative, since the cushion pressures can occasionally vary considerably from their mean values, as described later.

The skirt is considered to deflect vertically according to the local relative wave height only, hence each element of the skirt is assumed to be independent, and this corresponds closely to a finger skirt utilized by many ACVs. The elemental vertical skirt force is assumed to be proportional to the local cushion pressure, and is therefore given by

$$dR_v = \eta p dA_k \tag{7}$$

where dA_k is the element of skirt area deflected (seen in a side view). The constant η would effectively equal the tangent of the angle between the skirt and the vertical direction. The transverse stability skirt is straight and located at a distance ϵL ahead of the craft midpoint measured at the skirt hem, L being the length of the ACV.

We seek a solution to the craft response to a two-dimensional wave of the form

$$z_w = h_w \cos[k_w(x_w - c_w t)]$$

in which h_w is the wave amplitude, k_w its wave number, c_w its velocity, and x_w is measured in a stationary reference frame. The wave form may be re-expressed as

$$z_w = h_w \cos[k_w x - \omega t] \tag{8}$$

where ω is the encounter frequency.

Escape Areas

The escape areas under the skirt may be obtained by an integration along the skirt hem—assuming the craft position is known. The hem clearance is

$$e = z + x\theta - h_k(\mathcal{L}) - z_w \tag{9}$$

where x is measured forward from the craft center and h_k is the local skirt height, which is a function of the position along the skirt hem, $\mathcal{L}$. The discharge, or escape, area is then

$$A_e = \int eH(e)d\mathcal{L} \tag{10}$$

where H is the Heaviside step function. The integration is to be taken over the appropriate part of the skirt perimeter. In particular, the escape area under the stability skirt is

$$A_{e3} = BeH(e) \tag{11}$$

where B is the local beam, and it, together with e is evaluated at $x = \epsilon L$.

Skirt Contact Forces and Moments

Assuming the craft position, then the vertical force on the craft (positively upwards) due to skirt deflection will be

$$R_v = -\int p\eta eH(-e)d\mathcal{L} \tag{12}$$

(The restoring force on the stability skirt is assumed to be proportional to $p_2 - p_1$. Other models, such as an inflated bag skirt could easily be accounted for.)

To obtain the horizontal component of the skirt force, one may consider an element of deflected skirt lying on the wave surface with a local slope, α_w, given by

$$\alpha_w = \partial z_w/\partial x_w \mp -k_w h_w \sin(k_w x + \omega t) \tag{13}$$

Then the ratio of the elemental horizontal force (positively forward) to the vertical one is given by

$$\frac{dR_h'}{dR_v} - \frac{\alpha_w + \mu}{\mu\alpha_w - 1} \tag{14}$$

where μ is the local coefficient of friction. This sliding friction concept is difficult to apply to the case of motion on water, since the drag is dependent on the Reynolds number and many other factors. However, if the ratio of the local tangential to normal force on the water surface were taken as μ, then Eq. (14) would be valid.

Due to the slope of the skirt wall relative to the vertical, a vertical force is induced as noted above. In addition, the loss of skirt area during deflection (as seen sideways), results in an additional side force. This force acts normal to the skirt wall as seen in a plan view, and due to the symmetry of the craft about the x axis, will only give rise to a longitudinal force for those parts of the skirt which make a nonzero angle, β, relative to the x axis. This force is given by

$$dR_h'' = peH(-e)\sin\beta d\mathcal{L} \tag{15}$$

The horizontal force on the ACV due to skirt deflection is obtained from Eqs. (12 – 15)

$$R_h = R_h' + R_h'' = \int p\left[\sin\beta - \eta\frac{\alpha_w + \mu}{\mu\alpha_w - 1}\right]eH(-e)d\mathcal{L} \tag{16}$$

The moment on the vehicle (positively bow up) due to the skirt deflection is given by a similar integration, using the appropriate moment arms

$$M = \int p\Big[(z_g + h_k + \tfrac{1}{2}e)\sin\beta - \eta\Big\{x - x_g + (z_g + h_k + e)\frac{\alpha_w + \mu}{\mu\alpha_w - 1}\Big\}\Big]eH(e)d\mathcal{L} \tag{17}$$

Volume Flux Terms

Apart from the volume flow terms indicated by Eqs. (2–6),there are flow components due to pumping action by the motion of the vehicle, and by the motion of the exciting wave. The latter is given by

$$Q_w = \int \frac{\partial}{\partial t}(z_w)B(x)dx \qquad (18)$$

where the time differentiation is to be performed in the moving reference frame attached to the ACV. In fact, there are two wave pumping terms, one for each compartment. The longitudinal integration for each compartment is carried out over the appropriate length of the craft (ahead of, and behind, the stability skirt). Similarly, the pumping effect due to the motion of the vehicle is expressed by

$$Q_r = [\dot{z} + (\bar{x} - x_g)\dot{\theta}]S \qquad (19)$$

where $\bar{x}$ is the centroid of either compartment, and S is its area. Finally, the equation of continuity allows us to write

$$Q_{i1} = Q_{e1} + Q_{r1} - Q_{w1} + Q_{e3} \qquad (20)$$

$$Q_{i2} = Q_{e2} + Q_{r2} - Q_{w2} - Q_{e3} \qquad (21)$$

and

$$Q = Q_{i1} + Q_{i2} \qquad (22)$$

Equations of Motion

An application of Newton's equations of motion for the heave and pitch accelerations of the ACV gives

$$m(\ddot{z} + g) = p_1 S_1 + p_2 S_2 + R_v \qquad (23)$$

and

$$I\ddot{\theta} = p_1 S_1 (\bar{x}_1 - x_g) + \iota_{-} \Lambda_{-}$$

and

$$I\ddot{\theta} = p_1 S_1 (\bar{x}_1 - x_g) + p_2 S_2 (\bar{x}_2 - x_g) + M + \int p h_k (z_g + \tfrac{1}{2} h_k) \sin\beta d\mathcal{L} \qquad (24)$$

Numerical Solution

Rectangular Planform Example

Although most ACVs tend to have a rounded bow, we shall for the sake of simplicity, consider a rectangular craft whose base has an area L by B. Under this condition, the force component referred to in Eq. (15) will arise from the deflection of the stern, bow and stability skirts only (for which $\sin\beta = 1$).

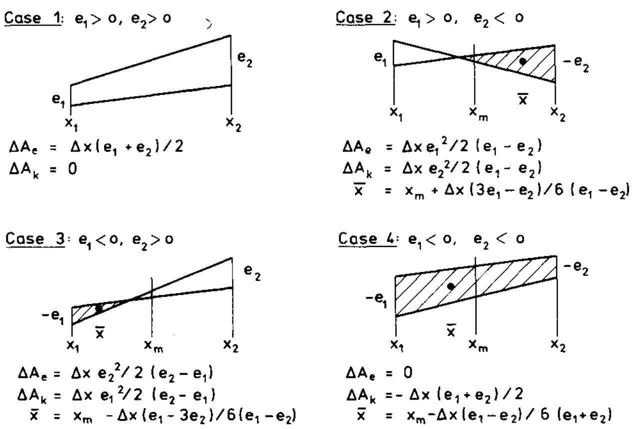

Fig. 2 Details of wave contact with element of skirt.

The volume fluxes due to wave pumping, Eq. (18), may be integrated analytically, using Eq. (8)

$$\begin{matrix} Q_{w1} \\ Q_{w2} \end{matrix} = \frac{h_w B\omega}{k_w} [\{\pm\cos(\tfrac{1}{2}k_w L) \pm \cos(\epsilon k_w L) \mp \cos(\omega t)$$
$$+ \{-\sin(\tfrac{1}{2}k_w L) \pm \sin(\epsilon k_w L)\}\sin(\omega t)] \qquad (25)$$

Iteration Procedure

Due to the nonlinear and interactive nature of the equations which have to be satisfied, no explicit solution may be realized. Instead, a numerical technique was utilized in which the vehicle was started instantaneously at $t = 0$ with a forward velocity c over the waves which travel at a speed c_w. It was found that a suitable set of initial conditions are

$$z = z_g + h_{k,\min} - h_w$$

and

$$\dot{z} = \ddot{z} = \theta = \dot{\theta} = \ddot{\theta} = 0 \qquad (26)$$

Thus, initially, both cushions are blocked by skirt contact with the wave. This provided a good starting point for the solution of the pressures p_1 and p_2 in Step 3, below.

The time was incremented in steps Δt. At each instant, the cushion pressures and skirt forces were computed; from these the accelerations in pitch and heave calculated, and an extrapolation to the next time step made. The procedure was continued over a sufficient number of waves (of period $T = 2\pi/\omega$) until the vehicle settled into a cyclic motion. The major steps in the program may be summarized as follows:

1) From the present estimate of craft position in heave and pitch, the escape areas, A_{e1}, A_{e2}, and A_{e3} [from Eq. (10), and the skirt contact areas, A_{k1}, A_{k2}, and A_{k3} and the moments of these areas about the craft center of gravity [using Eqs. (12), (16), and (17)], were computed. The integrals were approximated by a numerical integration around the skirt hem. The step size in this integration, $\Delta\mathcal{L}$, was obtained on the basis that one would need, say, a minimum of 20 points per craft length, and per wavelength, to define the skirt contact areas properly. A straight line interpolation was used, resulting in the elements of area being approximated by either triangles or trapezia. The four possible types of skirt element contact are shown in Fig. 2. For any of these, it may be shown that

$$\Delta A_e = \Delta\mathcal{L}(e_1 + e_2 + |e_1| + |e_2|)^2 / 4(|e_1| + |e_2|) \qquad (27)$$

$$\Delta A_k = \Delta A_e - \Delta\mathcal{L}(e_1 + e_2) \qquad (28)$$

and

$$\bar{x}_{\Delta A_k} = \tfrac{1}{2}(x_1 + x_2) + \Delta x(2e_1 - 2e_2 + |e_1| - |e_2|)/6(|e_1| + |e_2|) \qquad (29)$$

the last equation giving the longitudinal centroid of the element of skirt contact area. Here, the subscripts 1 and 2 refer to the two endpoints of the element.

2) The craft pumping terms Q_{r1} and Q_{r2} were calculated from the instantaneous craft velocities in heave and pitch. The wave pumping terms, Q_{w1} and Q_{w2}, were calculated from Eq. (25).

3) The instantaneous cushion pressures p_1 and p_2 were derived from Eqs. (1–6, 19–22, and 25). This was achieved by writing the two functions

$$u = \frac{2}{\rho_a}(A_{i1} C_{di1})^2 (p_f - p_1) - Q_{i1}|Q_{i1}|$$

and

$$v = \frac{2}{\rho_a}(A_{i2} C_{di2})^2 (p_f - p_2) - Q_{i2}|Q_{i2}| \qquad (30)$$

Thus Eqs. (2) and (3) are satisfied when $u=v=0$. The Newton-Raphson technique in two variables (see Ref. 22, p. 319), was used to solve this pair of equations. For this, one requires the four derivatives

$$\frac{\partial\mu}{\partial p_1} = \frac{2}{\rho_a}(A_{i1}C_{di1})^2[(\frac{\partial Q_{i1}}{\partial p_1}+\frac{\partial Q_{i2}}{\partial p_1})$$

$$\{C_{f2}+2C_{f3}(Q_{i1}+Q_{i2})\}-1]-2|Q_{i1}|\frac{\partial Q_{i1}}{\partial p_1}$$

$$\frac{\partial\mu}{\partial p_2} = \frac{2}{\rho_a}(A_{i1}C_{di1})^2\cdot(\frac{\partial Q_{i1}}{\partial p_2}+\frac{\partial Q_{i2}}{\partial p_2})$$

$$\{C_{f2}+2C_{f3}(Q_{i1}+Q_{i2})\}-2|Q_{i1}|\frac{\partial Q_{i1}}{\partial p_2} \quad (31)$$

$$\frac{\partial Q_{i2}}{\partial p_2} = \frac{2}{\rho_a}(A_{i_2}C_{di_2})^2\cdot(\frac{\partial Q_{i1}}{\partial p_1}+\frac{\partial Q_{i2}}{\partial p_1})$$

$$\{C_{f_2}+2C_{f_3}(Q_{i_1}+Q_{i_2})\}-2|Q_{i_2}|\frac{\partial Q_{i2}}{\partial p_1}$$

and

$$\frac{\partial v}{\partial p_2} = \frac{2}{\rho_a}(A_{i_2}C_{di_2})^2[(\frac{\partial Q_{i1}}{\partial p_2}+\frac{\partial Q_{i2}}{\partial p_2})$$

$$\{C_{f_2}+2C_{f_3}(Q_{i_1}+Q_{i_2})\}-1]-2|Q_{i_2}|\frac{\partial Q_{i2}}{\partial p_2}$$

in which, from Eqs. (20), (21), and (4-6)

$$\frac{\partial Q_{i1}}{\partial p_1} = A_{e_3}C_{de_3}/(2\rho_a|p_1-p_2|)^{1/2}$$

$$+A_{e_1}C_{de_1}/(2\rho_a|p_1|)^{1/2}$$

$$\frac{\partial Q_{i2}}{\partial p_1} = \frac{\partial Q_{i1}}{\partial p_2}$$

$$\frac{\partial Q_{i1}}{\partial p_2} = -A_{e_3}C_{de_3}/(2\rho_a|p_1-p_2|)^{1/2} \quad (32)$$

and

$$\frac{\partial Q_{i2}}{\partial p_2} = A_{e_3}/(2\rho_a|p_1-p_2|)^{1/2}$$

$$+A_{e_2}C_{de_2}/(2\rho_a|p_2|)^{1/2}$$

The Newton-Raphson iteration procedure then predicts a better solution to Eqs. (30) by increasing the current estimates of p_1, and p_2 by Δp_1 and Δp_2, respectively, where

$$\Delta p_1 = (v\frac{\partial\mu}{\partial p_2}-\mu\frac{\partial v}{\partial p_2})/\Delta$$

$$\Delta p_2 = (\mu\frac{\partial v}{\partial p_1}-v\frac{\partial\mu}{\partial p_1})/\Delta \quad (33)$$

and

$$\Delta = \frac{\partial\mu}{\partial p_1}\cdot\frac{\partial\mu}{\partial p_2}\cdot\frac{\partial v}{\partial p_1}$$

As a starting point, the cushion pressures at the previous time step were used. The accuracy criterion for convergence was that

$$\frac{|\Delta p_1|(S_1+S_2)}{mg} \text{ and } \frac{|\Delta p_2|(S_1+S_2)}{mg} < 10^{-4}$$

This degree of convergence was generally achieved in fewer than six iterations. The convergence was, however, poor under certain conditions.

These occurred when one of the cushion pressures was close to zero. Then the successive results of the iteration tended to oscillate about the final solution. If this behavior was detected after 65 iterations, then a simple mean of successive approximations was used. This fixup technique resulted in the condition (34) being satisfied.

4) The next step was to calculate the skirt contact forces and moments, from the cushion pressures, and contact areas, etc., mentioned previously in Steps 3 and 1.

5) The instantaneous heave and pitch accelerations were then computed using Eqs. (23) and (24).

6) The motion of the craft was extrapolated to the next point in time using the predictor technique

$$\dot z(t+\Delta t)=\dot z(t)+\Delta t\cdot\ddot z(t)$$

$$\dot\theta(t+\Delta t)=\dot\theta(t)+\Delta t\cdot\ddot\theta(t)$$

$$z(t+\Delta t)=z(t)+\tfrac{1}{2}\Delta t[\dot z(t)+\dot z(t+\Delta t)] \quad (35)$$

and

$$\theta(t+\Delta t)=\theta(t)+\tfrac{1}{2}\Delta t[\dot\theta(t)+\dot\theta(t+\Delta t)] \quad (36)$$

It was found that a suitable time step at a nondimensional frequency of $\omega(g/L)^{1/2}=4$ resulted in 64 points/cycle. The time step required was essentially independent of wave encounter frequency. It was also found that any time increment smaller than this resulted in practically the same craft motion. However, a larger step than this caused such a poor starting point for the Newton-Raphson technique in Step 3, that the wrong, or unnatural roots for the cushion pressures were found. This caused the timewise extrapolation procedure to diverge within a fraction of a cycle.

7) At the end of each cycle, the heave and pitch of the craft were compared to those during the previous cycle. The error criterion was based on the maximum changes between corresponding points from one cycle to the next, related to the total variation of the heave or pitch in one cycle

$$\frac{|\Delta z|\max}{z_{\max}-z_{\min}} \text{ and } \frac{|\Delta\theta|\max}{\theta_{\max}-\theta_{\min}} < 10^{-2} \quad (37)$$

Such convergence was generally achieved after only three cycles. That is, the damping of the system was so high that the motion in the third cycle was essentially the same as in the second one, occasionally even to an accuracy of 10^{-6} at the lower frequencies. At higher frequencies, corresponding to $\omega(L/g)^{1/2}=12$, up to six cycles were needed before condition (37) was satisfied.

8) After the completion of the calculation of the ACV motion, the maximum, minimum, average, and root mean square values of the following fourteen variables were obtained: z, $\dot z$, $\ddot z$, θ, $\dot\theta$, $\ddot\theta$, Q_{i1}, Q_{i2}, p_1, p_2, p_f, R_h, R_v, and M. (Due to the nonlinear craft response, one cannot, for example, relate time differentiations of z to powers of ω.)

In addition, a Fourier analysis of $z(t)$ and $\theta(t)$ was made. A Filon-Trapezoidal rule, as suggested by Tuck[23] was utilized for this purpose. The magnitude of, say the second harmonic, to the fundamental gives an additional measure of the nonlinearity of the response.

Encounter Frequency

Although the forcing wave was considered solid, account was taken of the usual velocity of propagation of water waves. Thus if

$$F=c/(gL)^{1/2} \quad (38)$$

$$F_w = c_w / (gL)^{1/2} \tag{39}$$

and

$$k_w = 2\pi / L_w = 1/L \cdot F_w^{\,2} \tag{40}$$

where L_w is the wavelength, then the following relation between wave speed and encounter frequency may be derived

$$F_w = \{ -1 \pm (1 + 4F[\omega(L/g)^{1/2}])^{1/2} \} / 2[\omega(L/g)^{1/2}] \tag{41}$$

In general there are four different wave speeds that result in the same encounter frequency (if both positive and negative valued are considered). If $4\omega F > 1$, there are only two possible wave speeds.

Results

Numerical investigations were carried out for the craft, whose particulars appear in Table 1. The case of head seas ($F_w < 0$) only was studied. This, incidentally, yields a unique value of the wave speed in terms of the encounter frequency.

Figure 3 illustrates the heave and pitch response as a function of wave height. Maximum, minimum and root-mean-square values are presented. These are related to the mean value during the cycle of the motion, and divided by the wave height. The pitch angle is, instead, divided by the wave slope. In order to make a direct comparison between these measures of the craft motion, the root-mean-square parameters are multiplied by $(2)^{1/2}$. In a linear theory, the maximum, minimum, and root-mean-square values (as defined here) would be the same, and this is seen to be the case for heave as $h_w/L \to 0$. Nonlinearity is indicated by a spread of these parameters as h_w/L increases, and also by the mere fact that they are not constant.

Table 1 Craft particulars

Dimensional variable			Dimensionless variable	
L	= 30.0	m	Base Unit	
g	= 9.80	m/s^2	Base Unit	
ρ	= 1.025	t/m^3	Base Unit	
B	= 15.0	m	B/L	= 0.5
m	= 166.0	t	$m/\rho L^3$	= 0.006
I	= 8.10×10^6	kgm^2	$I/\rho L^5$	= 3.25×10^{-4}
x_g	= 0.0	m	x_g/L	= 0.0
z_g	= 3.00	m	z_g/L	= 0.1
h_{k_1}	= 2.40	m	h_{k_1}/L	= 0.08
h_{k_2}	= 2.40	m	h_{k_2}/L	= 0.08
h_{k_3}	= 2.40	m	h_{k_3}/L	= 0.08
C_{f_1}	= 12.05	kPa	$C_{f_1}/\rho gL$	= 0.04
C_{f_2}	= 0.0	kg/sm	$C_{f_2}(L^3/g\rho^2)^{1/2}$	= 0.0
C_{f_3}	= -0.0380	kg/m^4	$C_{f_3}L^4/\rho$	= -30.0
A_{i_1}	= 4.50	m^2	A_{i_1}/L^2	= 0.005
A_{i_2}	= 4.50	m^2	A_{i_2}/L^2	= 0.005
ρ_a	= 1.280	kg/m^3	ρ_a/ρ	= 0.00125
ϵ	= 0.0		no change	
η_1	= 1.0		"	
η_2	= 1.0		"	
η_3	= 0.0		"	
μ	= 0.0		"	
C_{di_1}	= 1.0		"	
C_{di_2}	= 1.0		"	
C_{de_1}	= 0.55		"	
C_{de_2}	= 0.55		"	
C_{de_3}	= 0.6		"	

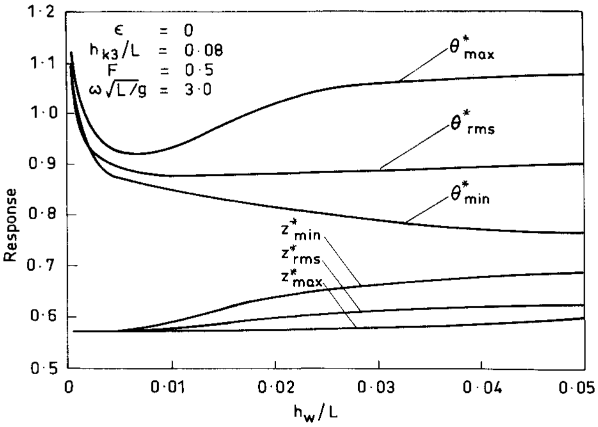

Fig. 3 Craft response as a function of wave height.

On the other hand, the pitch is seen to behave in a strongly nonlinear way as $h_w/L \to 0$, and is seen to (apparently) approach infinity. This phenomenon is no doubt due to the quadratic nature of the pressure-flux equation for flow under the stability skirt, as discussed in the Introduction.

The variation of these parameters with wave-encounter frequency is shown in Fig. 4, a–f, for various Froude numbers and modifications to the stability skirt depth (h_{k3}), and position (ϵL). Passing from Fig. 4 a–c, illustrates the effect of changing the Froude number. Weak resonant peaks are indicated, particularly for the heave motion. More of these peaks are evident at the lower Froude numbers, due to the wavelengths being shorter in comparison to the craft.

Of further interest is the manner in which the response curves for the different wave heights cross each other. The behavior as $\omega(L/g)^{1/2} \to 0$ is not obvious without further calculation. It appears that heave response may approach unity, but the pitch response may not. At low encounter frequency, the nonlinearity with respect to wave height diminishes.

Figures 4d and e illustrate the effect of shortening the stability skirt. It is seen that a peak response in pitch has developed in the region of $\omega(L/g)^{1/2} \doteqdot 5.5$. For a stability skirt height of 0.75 times the general skirt height (Fig. 4e), the worse pitch response is approximately double that when the stability skirt is equally deep. The stability skirt has been shifted aft $0.1L$ in Fig. 4f. The response curves are very similar to those in Fig. 4a, indicating the minor role this parameter plays.

The phase angles (ahead of the incident wave) of the fundamental components of the response are shown in Fig. 5 (corresponding to Fig. 4a). A strong dependence on wave height is again noted. For example, at $\omega(L/g)^{1/2} = 8$, the heave motion leads the wave by 4° for $h_w/L = 0.01$, but lags it by 40° for $h_w/L = 0.03$. The ratio of the second harmonic to the fundamental is given in Fig. 6. At low frequencies the pitch motion is mainly at the fundamental but at the high frequency end, the heave motion differs more from simple harmonic motion. Figure 7 displays second harmonic content in the two cushion pressures. This is generally much higher, in comparison to the results in Fig. 6. (The pressure harmonics are of the order of n^2 greater than the motion harmonics, where n is the number of the harmonic.)

Figures 8 and 9 present the maximum excursions of the cushion pressures during the motion. These almost always increase with wave height (as one would expect, since they are not nondimensionalized against wave height). They also generally increase with frequency. Since the average dimensionless cushion pressure is very close to $m/\rho BL^2$ (=0.012 here), it is seen from Fig. 9 that the cushion pressures can become negative, relative to atmospheric pressure, at the higher frequencies. This is presumably a consequence of the

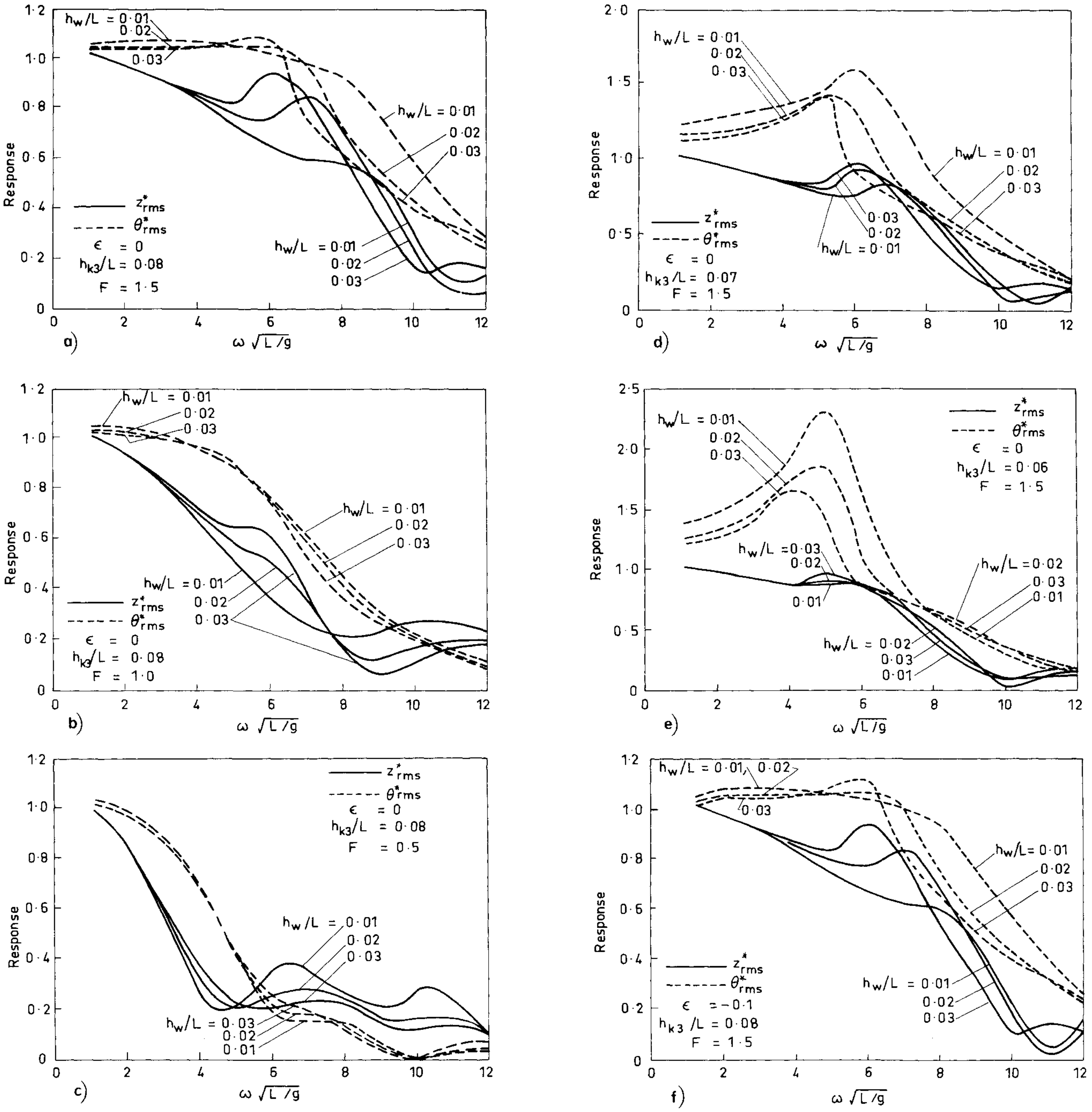

Fig. 4 Craft response as a function of encounter frequency: a) $F=1.5$, $h_{k3}=0.08$, $\epsilon=0$; b) $F=1.0$, $h_{k3}=0.08$, $\epsilon=0$; c) $F=0.5$, $h_{k3}=0.08$, $\epsilon=0$; d) $F=1.5$, $h_{k3}=0.07$, $\epsilon=0$; e) $F=1.5$, $h_{k3}=0.06$, $\epsilon=0$; f) $F=1.5$, $k_{k3}=0.08$, $\epsilon=-0.1$.

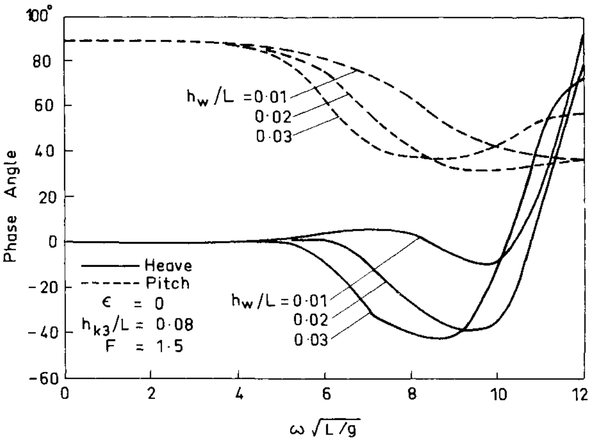

Fig. 5 Phase angles of craft response (ahead of incident wave).

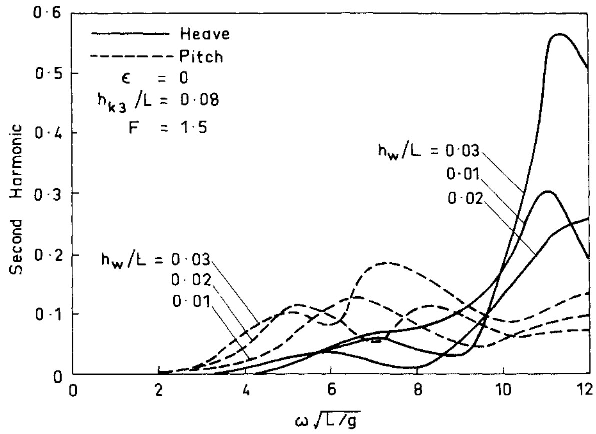

Fig. 6 Second harmonic components in craft response (related to the fundamental).

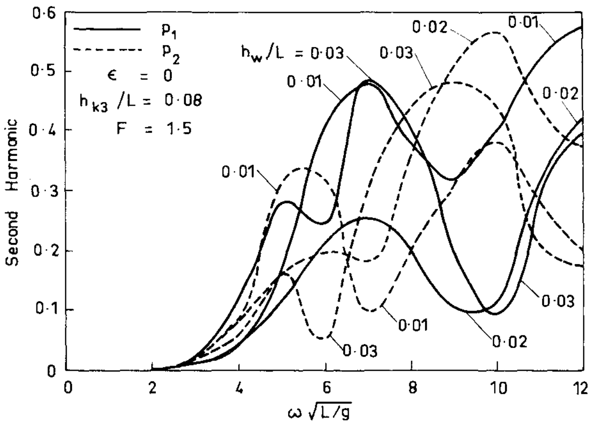

Fig. 7 Second harmonic components in cushion pressures (related to the fundamental).

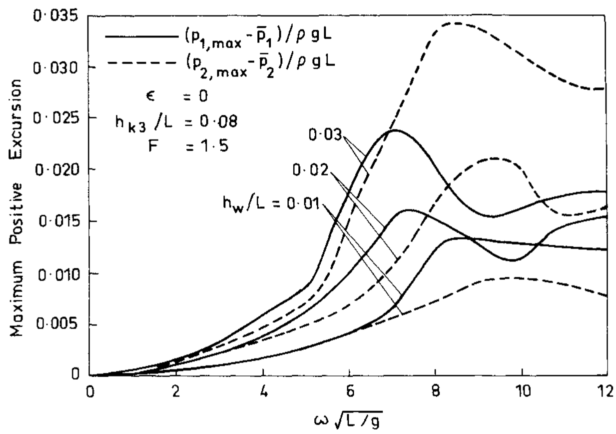

Fig. 8 Maximum positive excursions in cushion pressures.

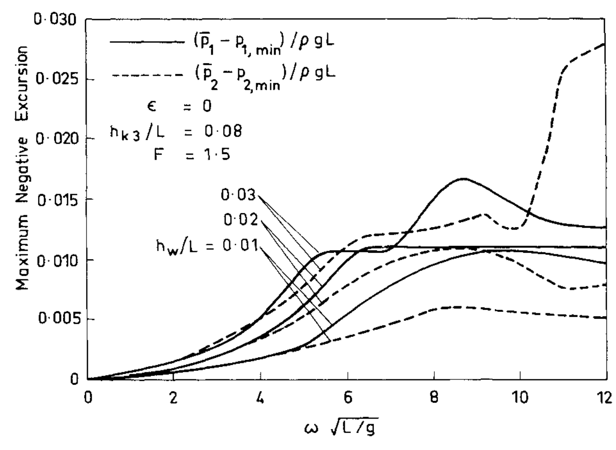

Fig. 9 Maximum negative excursions in cushion pressures.

assumption of incompressible flow. No doubt, if compressibility were allowed, the magnitude of the pressure excursions would be diminished. Finally, an example of the skirt contact forces and moments is shown in Fig. 10. For this case, no skirt contact occurs for $\omega(L/g)^{1/2}<3$. These forces and moments are generally only a few percent of those generated by the cushion pressures, and therefore hardly contribute to the motion of the craft.

Conclusions

Present Work

Basically, the calculations presented in this paper have confirmed the nonlinear nature of the craft response. However, it is interesting to note that for typical wave amplitudes, say $0.01<h_w/L<0.03$, the unit response is sufficiently constant, that in a random sea, the craft response may possibly be obtained by a linear superposition technique. This would certainly simplify practical calculations. However, this discussion is purely conjecture, and such a scheme would have to be verified.

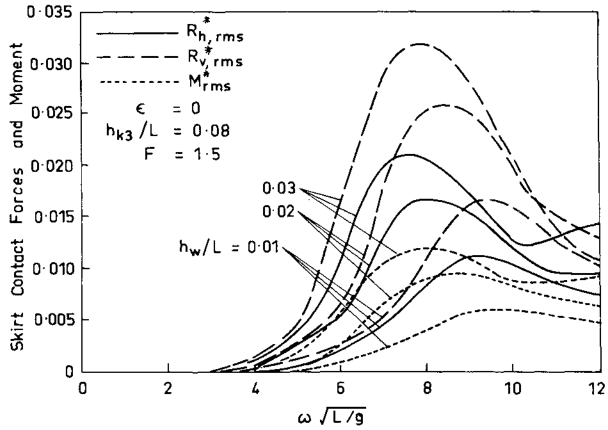

Fig. 10 Skirt contact forces and moment.

The skirt contact forces were found to be sufficiently negligible so that, with hindsight, it may be said that the skirt friction coefficient, μ, may indeed, be put to zero, with little effect on the craft motion. The pressure fluctuations were great enough at the higher frequencies to cast some doubt on the results under those conditions. There is an indication that compressibility then plays a role.

Future Work

Computational experiments can be extended in various paths. One point of interest would be to complete the results in the low frequency range. The effect of compressibility should also be included. This would modify some of the equations, but the overall technique would be unaltered. It should be pointed out here, that the negative pressures, referred to in the results, only occurred during a small fraction of the motion cycle. In that short time interval, the skirt system may not have time to collapse, and hence might even be an observable phenomenon. Involving somewhat greater work, would be the consideration of various craft planforms, including a rounded bow. Finally, as pointed out in the Introduction, the compliance of the water surface has been ignored, and the effect of this should be examined within the framework of linearized potential flow theory.

References

[1] Swaan, W.A. and Wahab, R., "The Behaviour of a Ground Effect Machine Over Smooth Water and Over Waves," Vol. 4, *Hovering Craft and Hydrofoil,* May 1965, pp. 18–24.

[2] Dyne, G., "An Investigation Concerning the Motions of a Hovercraft in Regular Head Seas," *Transactions of the Royal Institution of Naval Architecture,* Vol. 110, 1968, pp. 465–477.

[3] Van Den Brug, J.B., "The Dynamic Stability of a Ground Effect Machine Model, Determined by Static-and Forced-Oscillation Tests," *Proceedings of the 7th Symposium on Naval Hydrodynamics,* 1968,

[4] Van Den Brug, J.B. and Van Staveren, P., "An Experimental Study on the Dynamic Stability of a Ground Effect Machine Model in Heave, Pitch and Roll," *International Shipbuilding Progress,* Vol. 16, Sept. 1969, pp. 277–297.

[5] Murthy, T.K.S., "Motions of Marine Craft in a Wind Generated Seaway," *Proceedings of the 7th Symposium on Naval Hydrodynamics,* Panel Discussion No. 3, 1968,

[6] Stanton Jones, R., "Hovercraft Skirt Development—an Engineering and Performance Review," *Transactions of the Royal Institution of Naval Architecture,* Vol. 110, Oct. 1968, pp. 499–524.

[7] Lin, J.D., "Dynamic Behaviour of GEMs in Motion over Waves," *Journal of Ship Research,* Vol. 6, April 1963, pp. 1–10.

[8]Ozawa, S., "Spring Constants of the Peripheral-Jet Air Cushion Vehicle in Heaving Motion," *Journal of Japan Society for Aeronautical and Space Sciences,* Vol. 18, Feb. 1970, pp. 69–73. In Japanese.

[9]Hogben, N., "Hovering Craft Over Water," Tech. Memo. 119, 1966, National Physical Lab., Ship Division, England.

[10]Reynolds, A.J., "A Linear Theory for the Heaving Response of a Hovercraft Moving Over Regular Waves," *Journal of Mechanical Engineering Science,* Vol. 14, April 1972, pp. 147–150.

[11]Reynolds, A.J., West, R.P., and Brooks, B.E., "Heaving and Pitching Response of a Hovercraft Moving over Regular Waves," *Journal of Mechanical Engineering Science,* Vol. 14, Oct. 1972, pp. 340–352.

[12]Yamamoto, A., "Nonlinear Heaving Motion of Plenum-Chamber-Type Air Cushion Vehicles," *Journal of Japan Society for Aeronautical and Space Sciences,* Vol. 18, Feb. 1970, pp. 73–78 (in Japanese).

[13]Leatherwood, J.D., Dixon, G.V., and Stephens, D.G., "Heave Response of a Plenum Air Cushion Including Passive and Active Control Concepts," TN D-5202, 1969, NASA.

[14]Guienne, P., "Air Cushion Vehicle Suspension," *Aeronautique et l'Astronautique,* No. 24, Aug. 1970, pp. 17–28 (in French).

[15]Leatherwood, J.D., "Analog Analysis of the Heave Response and Control of a Plenum-Type Air-Cushion Vehicle," TN D-6257, 1971, NASA.

[16]Genin, J., Ginsberg, J.H., and Ting, E.C., "Fluid Suspension Model for Air-Cushioned Vehicles," *Journal of Sound and Vibration,* Vol. 25, Nov. 1972, pp. 83–93.

[17]Bickford, L.L. and Olson, G.K., "Dynamics of a Multiple-Skirt Air-Cushion Vehicle," *Journal of Aircraft,* Vol. 6, Nov.-Dec. 1969, pp. 564–565.

[18]Breslin, J.P., "The Dependence of SES Heave Response on Model Scaling Paremeters as Deduced from a Simplified Linear Approach," Tech. Memo. 157, 1969, Davidson Lab., Stevens Institute of Technology, Hoboken, N.J.

[19]Richardson, J.R., "A Scaling Technique for Hovercraft Models," Rept. No. 13, 1970, National Physical Lab., Hovercraft Unit, England.

[20]Trillo, R.L., *Marine Hovercraft Technology,* Leonard Hill, London, 1971, pp. 108-122.

[21]"Fans," *Guide and Data Book—Fundamentals and Equipment,* Ch. 40, American Society of Heating, Refrigeration and Air-Conditioning Engineers, New York, 1961, pp. 631–640.

[22]Carnahan, B., Luther, H.A., and Wilkes, J.O., *Applied Numerical Methods,* Wiley, New York, N.Y., 1969, p. 319.

[23]Tuck, E.O., "A Simple 'Filon-Trapezoidal' Rule," *Mathematics of Computation,* Vol. 21, April 1967, pp. 239–241.